Program Cover Document --- STA 410: Mathematical Statistics

I. Basic Course Information

STA 410—Mathematical Statistics is primarily a junior/senior level course. It is a required course for MATC majors, and an option for MATA and MATT majors and both Mathematics and Statistics minors. It is scheduled for two 80-minute meetings each week. Its prerequisites are STA 215: Statistical Inference, and MAT 316: Introduction to Probability.

II. Learning Goals

The primary goal of STA 410 is to give students a firm foundation in the theory of statistics. In STA 215, students are presented with theorems and formulae that they learn to use; in STA 410 they learn to prove these theorems and derive all of the formulae. This course will introduce students to inference techniques beyond those learned in previous classes as well because they will learn to apply the techniques to samples taken from distributions that are not approximately normal.

Statistical inference deals with using information taken from random samples to draw conclusions about the populations that the samples came from, along with some type of measure of how "good" those conclusions are. Different samples will contain different observations and hence will give different estimates of population parameters. To know how close the sample statistics will be to the population parameters, it is necessary to know what values different samples can give. This information is contained in the statistic's sampling distribution, and most of classical statistical inference is based on sampling distributions. Hence a major part of the course will be devoted to deriving the sampling distributions of the most commonly used statistics. To do this will require the students to learn methods of transformations of variables—the method of distribution functions, the method of moment generating functions, and the method of transformations. These can then be used to derive the most commonly used statistical distributions—the t, F, and χ^2 distributions and the distributions of order statistics—as well as proving theorems such as the Central Limit Theorem.

Statistical inference is broken into two major categories—estimation and hypothesis testing. Furthermore, estimation is broken down into point estimates and interval estimates, i.e. confidence intervals. Students will learn methods for deriving point estimators, including the method of moments and the method of maximum likelihood. They will then learn how to compare estimators using such properties as bias, Mean Squared Error, consistency, and sufficiency, and how to derive good estimators such as the Minimum Variance Unbiased Estimator. They will learn to derive confidence intervals for parameters from a variety of different distributions using the pivotal method, and how to derive hypothesis tests using the likelihood ratio method. They will learn to calculate Type I and Type II Errors for hypothesis tests, and compare them using the power. Optionally, some Bayesian methods may also be introduced.

The Mathematical Statistics course is designed to meet the needs of all the students in the class. The course will give the necessary knowledge of statistical theory needed for statistics majors who are planning to get a job after graduation, as well as those who intend to continue on to graduate school. Non-statistics majors will also benefit from learning more complex mathematical models and learning to do high-level reasoning. Also, mathematical statistics is a required subject in actuarial science. This course builds on the methods learned in probability, which is the main subject of the first actuarial exam, and students planning to pursue an actuarial career should be well prepared for the first actuarial exam after completing this course.

An additional learning goal of Mathematical Statistics is the further development of both a student's ability to assess the reasonableness of answers and a student's ability to read, write, and understand proofs.

III. Student Assessment

This course is intended to be highly homework intensive. Weekly problem and proof assignments will constantly provide students with opportunities to show their understanding of the material. At the same time students will receive weekly feedback on their work and their progress. A combination of assignments, quizzes, and tests throughout the course will provide further valuable information both for the instructor and the individual students.

IV. Learning Activities

The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.

Departmental Course Syllabus --- STA 410: Mathematical Statistics

Introduction: A typical syllabus for Mathematical Statistics follows this sheet. Any syllabus for Mathematical Statistics should include the points listed below.

I. Basic information on course and instructor

A. Purpose statement: Mathematical Statistics introduces students to the underlying theory of statistics. In this course they will learn to prove all of the theorems and derive all of the formulas that they learned to use in their introductory statistics courses. To do this they will need to learn probability transformation techniques to derive the sampling distributions of the most commonly used statistics. They will also learn the theory of point estimation, interval estimation, and hypothesis testing, which will allow them to do statistical inference when sampling from populations that do not have an approximately normal distribution.

B. Course description: An introduction to the theory of statistics. Topics include transformations of variables; sampling distributions of common statistics; method of moments and maximum likelihood estimation; properties of point estimates including bias, MSE, consistency, and sufficiency; confidence intervals; and hypothesis testing, including Type I and II Errors and power.

C. Course prerequisites: STA 215 and MAT 316.

II. Learning goals

A. Content goals: Students will gain proficiency with many basic topics in the theory of statistics. Students will learn to derive sampling distributions and prove theorems by using transformation of variables techniques, including the CDF, MGF, and transformation methods. The distributions of order statistics will also be learned. They will learn the theory of point estimation, being able to derive different point estimators for parameters, including the MVUE, and compare them using different properties. They will also be able to form confidence intervals for a variety of distribution parameters using the pivotal method, and derive hypothesis tests using the likelihood ratio technique. They will be able to compare hypothesis tests using Type I and II errors and power.

B. Performance goals: At the completion of the course, students should demonstrate competence with the theory of statistics. They should be able to prove all the major theorems that they have learned both in Statistical Inference (STA 215) and in Probability (MAT 316). They should be able to derive point and interval estimates for populations modeled by any type of distribution, including ones that require the use of order statistics. They should know the basics of hypothesis testing, including how to derive test statistics and compare different ones using power.

III. Student assessment

A. Assessment Plan: This course is intended to be highly homework intensive. Weekly problem and proof assignments will constantly provide students with opportunities to show their understanding of the material. At the same time students will receive weekly feedback on their work and their progress. A combination of assignments, quizzes and tests throughout the course will provide further valuable information both for the instructor and the individual students. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.

B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.

C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: The specific choices of learning activities will depend upon the instructor, but it is expected that they will consist of some combination of lectures, group work, student presentations, individual homework, quizzes, tests and final exam.

B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of sampling distributions, estimation, and hypothesis testing will be given equal emphasis.

C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of mathematical statistics and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students receive continual regular feedback on their work.

STA 410 -- MATHEMATICAL STATISTICS
Fall 2004

Instructor: Dr. Sharon Navard

Office: P244 Science Complex **Phone**: 771-3040 (Office) 883-8393 (Home)

In Class: TBD

Office Hours: TBD; Other times by appointment.

E-mail: navard@tcnj.edu

Course Description: In STA 215, you were presented with theorems and formulas that you learned to use; in this class you will learn to prove those theorems and derive all of the formulas. This course will introduce you to inference techniques beyond those learned in previous classes as well because you will learn to apply the techniques to samples taken from distributions that are not approximately normal.

Statistical inference deals with using information taken from random samples to draw conclusions about the populations that the samples came from, along with some type of measure of how "good" those conclusions are. Different samples will contain different observations and hence will give different estimates of population parameters. To know how close the sample statistics will be to the population parameters, it is necessary to know what values different samples can give. This information is contained in the statistic's sampling distribution, and most of classical statistical inference is based on sampling distributions. Hence a major part of this course will be devoted to deriving the sampling distributions of the most commonly used statistics. To do this you will need to learn methods of transformations of variables—the method of distribution functions, the method of moment generating functions, and the method of transformations. These can then be used to derive the most commonly used statistical distributions—the t, F, and χ^2 distributions and the distributions of order statistics—as well as proving theorems such as the Central Limit Theorem.

Statistical inference is broken into two major categories—estimation and hypothesis testing. Furthermore, estimation is broken down into point estimates and interval estimates, i.e. confidence intervals. You will learn methods for deriving point estimators, including the method of moments and the method of maximum likelihood, and then learn how to compare estimators using such properties as bias, Mean Squared Error, consistency, and sufficiency, and how to derive good estimators such as the Minimum Variance Unbiased Estimator. You will learn to derive confidence intervals for parameters from a variety of different distributions using the pivotal method, and how to derive hypothesis tests using the likelihood ratio method. Finally, you will learn to calculate Type I and Type II Errors for hypothesis tests, and compare them using the power.

Textbook: *Mathematical Statistics with Application*, 6th Ed., by Wackerly, Mendenhall, and Scheaffer.

Homework: Homework problems will be assigned regularly. These will not be collected and graded; however, the solutions to these will be put on reserve in the library. The third weekly class meeting will be the time that I will spend answering questions on the homework. Before any problems are covered in class, I expect students to have worked them and looked at the solutions. Students are encouraged to form study groups to work on these problems. Quizzes will also be given--some will be in-class and others take-home; students may not work together on these. No take-home quizzes will be accepted late. If you want to do well on the quizzes and tests, I recommend spending a considerable amount of time on the homework.

Tests and Grading: You will have a closed-book quiz every Wednesday, covering all material covered the previous week (or since the last quiz). Any changes to this quiz schedule will be announced in class. At times I may ask you to review material from your calculus, statistics, or probability classes that I think you may be rusty on. Any time I ask you to review something, you may have a short open-book quiz on it the next class period. Approximately two in-class tests will be given. It is possible that parts of these will be take-home, and a third take-home test may be given. No test grades will be dropped and no makeup tests will be given. If you cannot attend on the day of the test, make arrangements to take the test early. No tests will be given after the tests are returned to the class. Also, a comprehensive final exam will be administered at the end of the semester. Any legitimately missed tests will be made up on the final. Any student caught cheating on tests or quizzes will be subjected to the strongest possible disciplinary action in accordance with the TCNJ Academic Integrity Policy. The grade in the course will be determined as follows:

Tests:	50%
Quizzes:	20%
Final Exam:	30%.

STA 410 Tentative Coverage
Fall 2004

Class	Date	Material Covered
1	Aug. 28	Review of Statistics
2	Sept. 3	Review of Probability (Chapter 5)
3	Sept. 4	Sec. 6.1-6.3 -- Method of Distribution Functions
4	Sept. 9	Sec. 6.4-- Method of Transformations
5	Sept. 11	Sec. 6.6—Two-variable Transformations
6	Sept. 16	Sec. 6.5 -- MGF Method
7	Sept. 18	Sec. 6.7 -- Order Statistics
8	Sept. 23	Sec. 7.1 -- Sampling Distributions
9	Sept. 25	Sec. 7.2 -- Sampling Dist'ns Related to the Normal
10	Sept. 30	Sec. 7.2 -- χ^2, t, and F Distributions
11	Oct. 2	Sec. 7.3-4 -- Central Limit Theorem
12	Oct. 7	Sec. 8.1-8.2 -- Bias of Point Estimators
13	Oct. 9	Test 1
14	Oct. 14	Sec. 8.3 -- MSE of Point Estimators
15	Oct. 16	Sec. 8.5 -- Confidence Interval Theory
16	Oct. 23	Sec. 8.6--8.9 -- Confidence Interval Applications
17	Oct. 28	Sec. 9.1-9.3 -- Relative Efficiency and Consistency
18	Oct. 30	Sec. 9.4-9.5 -- Sufficiency and MVUE's
19	Nov. 4	Sec. 9.6 -- Method of Moments
20	Nov. 6	Sec. 9.7-9.8 -- Maximum Likelihood Estimators
21	Nov. 11	Sec. 10.1-2; 10.7 -- Introduction to Hypothesis Testing
22	Nov. 13	Sec. 10.4 -- Type II Error Probabilities
23	Nov. 18	Test 2
24	Nov. 20	Sec. 10.9 -- Hypothesis tests for variances
25	Nov. 25	Sec. 10.7, 10.10 -- Power of Tests
26	Dec. 2	Sec. 10.11 -- Likelihood Ratio Tests
27	Dec. 4	Chap. 14 -- Chi Squared Tests
28	Dec. 9	Review; Evaluations