Program Cover Sheet --- MAT 351: Geometry

I. Course Information

MAT 351: Geometry is normally a junior/senior level course. It meets for two 80-minute lecture periods. The prerequisites are MAT 200 and MAT 229 or permission of the instructor.

Geometry, derived from the Greek for "measurement of the earth," is the study of shapes and their measurements. The ancient Greeks devoted much study to the geometry of the flat plane – an abstract generalization of a farmer's field. The theorems of the ancient Greeks were collected by Euclid in his 13 volume work *The Elements*, where he developed many of the theorems of what we now know as Euclidean geometry. Most of what is learned in a modern day high school geometry class can be traced back to the ancient Greeks. It was only in the mid-nineteenth century that mathematicians realized that other, equally valid two-dimensional geometries existed. This course will further develop students' knowledge of the geometry of Euclid and expose students to non-Euclidean geometry.

II. Learning Goals

Geometry is a course designed primarily for mathematics education majors. Therefore, one of the fundamental goals of the course is to bring students to a level of familiarity with geometric concepts that is more than sufficient for teaching high school geometry courses. This course is also an excellent course for liberal arts mathematics majors who wish to prepare for further study in graduate school or who simply wish to be exposed to an area of mathematics not addressed in the introductory courses in our department.

In this course, students will be exposed to both Euclidean and non-Euclidean geometries from a variety of mathematical perspectives. In addition to helping students understand the specific geometries involved, this will call upon students to think about a single mathematical object in a variety of ways, both abstract and concrete. Thus, in the process of becoming a competent undergraduate geometer, the student will continue to develop as a mathematician.

This course will entail a highly abstract treatment of those geometric objects with which students consider themselves familiar. This sense of familiarity will be both a help and a hindrance for students. On the one hand, students will be able to build upon concepts with which they are comfortable as opposed to learning an entirely new domain of discourse. On the other hand, the Euclidean Geometry that students learn in high school is only one of the several geometries, each with its own assumptions and metrics that will be covered in this course. Therefore, students will need to learn the limits of the applicability of their previous exposure to geometry.

Students will grow in their ability to reason abstractly, to read mathematics and write proofs, and to find a mature mathematician's balance between the abstract and the concrete. Drawing upon geometry learned in high school as well as analytic techniques learned in calculus and introducing and applying the algebraic notion of a group, the course will help students learn to integrate bits of mathematics learned in various settings over the years.

III. Assessment

Students will be expected to demonstrate both technical and conceptual competence in geometry. Exams and homework assignments will directly address the key performance goals of the course as listed in the course syllabus guide. These performance goals require students to think abstractly, to apply abstract concepts in concrete settings, and to utilize knowledge gained in previous courses. Students will be expected to write proofs throughout the course, both in homework assignments and on exams. Students will receive written feedback on their homework and exams, which will help the students continue to develop as mathematicians.

IV. Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects. These activities will address most of the general goals of the liberal arts mathematics program as well as non-professional goals of the mathematics education program. In particular, students will continue learning to read, write, and explain mathematics, to think logically and abstractly, and to apply knowledge learned in previous courses to solve problems in this course.

Departmental Course Syllabus -- MAT 351: Geometry

In this course, we will explore Euclidean, hyperbolic and elliptic geometries from a variety of viewpoints, both ancient and modern. Students will become well-versed in geometry at the undergraduate level. This course will prepare the liberal arts student for graduate study in geometry and topology, and it will provide mathematics education students with a more-than-sufficient background for teaching a course in high school geometry.

Multivariable Calculus and Discrete Mathematics are prerequisites for the course. The notion a line integral and computations of area involving double integrals will arise. Students will be expected to read mathematics well and to write proofs competently. The course will also draw on notions learned in high school geometry.

Learning Goals

<u>Content</u>

Students in this course will gain acquaintance with both Euclidean and non-Euclidean geometry. Each geometry will be understood from three different perspectives: first, as the collection of theorems following from a particular set of axioms; second, as the two dimensional geometry arising from a particular metric; and third, as the geometry obtained from a set together by specifying the group of rigid motions on the set. At least one of these viewpoints will be developed in detail.

Many students will enter the course with the misconception that all geometry is Euclidean and will mistakenly assume Euclidean theorems in a non-Euclidean setting. Students will learn which familiar geometry theorems hold in the non-Euclidean geometries as well as Euclidean geometry and which are specific to Euclidean geometry. Students will come to understand that these geometries are equally valid yet different, and that each is a useful tool in an appropriate setting.

The student will become facile in working with the geometric objects in a two-dimensional geometry, including lines, polygons, and circles. The notions of congruence and parallelism will be explored in depth.

<u>Performance</u>

The successful geometry student will be able to do all of the following:

1. Characterize Euclidean and non-Euclidean geometries through
    a. an axiomatic description
    b. a description of the appropriate metric
    c. a description of the rigid motions of the geometry.
2. Recognize a geometry given any of the above characterizations.
3. Compute lengths and areas using a metric.
4. Demonstrate proficiency with rigid motions.
5. Determine congruence or non-congruence of different objects in a particular geometry.
6. Categorize lines in a particular geometry according to their parallelism properties.
7. Prove or disprove theorems in the various geometries.
8. Identify which familiar theorems from Euclidean geometry hold in a particular non-Euclidean setting.

Assessment

Students will be assessed through some combination of graded homework assignments, oral communication, group and individual projects, and in-class and final examinations. The homework assignments and examinations will directly assess the eight performance goals listed above. Formal and informal oral communication provides students with an alternative route for demonstrating competence with one or more of the performance goals and for developing geometric intuition interactively with fellow students and the instructor.

Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.

Learning Activities

At the discretion of the instructor, learning activities will include any or all of the following: attendance at lectures offered by the instructor and other students, written homework assignments, oral presentations, participation in classroom discussions, and group or individual projects.

The course outline will vary according to the preferences of the instructor. However, the outline must address the learning goals given above. Moreover, elliptic geometry should not be the primary focus of the course, and Euclidean rigid motions should be given a thorough treatment. A sample course outline is provided in the attached Fall, 2003 syllabus.

Syllabus
Math 351
Fall, 2003
Prof. Cynthia Curtis

Text: *The Poincare Half-Plane: A Gateway to Modern Geometry* by Saul Stahl

Reaching me and getting help: My office is SC P-208, and my extension is x2026. My email address is ccurtis@tcnj.edu. Information about the course will be available on SOCS. I will have regularly scheduled office hours on Tuesdays from 12:30pm to 1:50 pm and on Thursdays from 11:00 am to 12:20 pm. You may also make an appointment with me for another time.

In this course, we will explore Euclidean, hyperbolic and elliptic geometries from a variety of viewpoints, both ancient and modern. You will become well-versed in geometry at the undergraduate level. This course will prepare the liberal arts student for graduate study in geometry and topology, and it will provide mathematics education students with a more-than-sufficient background for teaching a course in high school geometry.

Multivariable Calculus and Discrete Mathematics are prerequisites for the course. The notion a line integral and computations of area involving double integrals will arise. You will be expected to read mathematics well and to write proofs competently. The course will also draw on notions learned in high school geometry.

Learning Goals

Content

In this course, you will gain acquaintance with both Euclidean and non-Euclidean geometry. Each geometry will be understood from three different perspectives: first, as the collection of theorems following from a particular set of axioms; second, as the two dimensional geometry arising from a particular metric; and third, as the geometry obtained from a set together by specifying the group of rigid motions on the set.

Many of you will enter the course with the misconception that all geometry is Euclidean and will mistakenly assume Euclidean theorems in a non-Euclidean setting. One of your goals will be to learn which familiar geometry theorems hold in the non-Euclidean geometries as well as Euclidean geometry and which are specific to Euclidean geometry. You will come to understand that these geometries are equally valid yet different, and that each is a useful tool in an appropriate setting.

You will become facile in working with the geometric objects in a two-dimensional geometry, including lines, polygons, and circles. The notions of congruence and parallelism will be explored in depth.

Performance

The successful geometry student will be able to do all of the following:

1. Characterize Euclidean and non-Euclidean geometries through
    a. an axiomatic description
    b. a description of the appropriate metric
    c. a description of the rigid motions of the geometry.
2. Recognize a geometry given any of the above characterizations.
3. Compute lengths and areas using a metric.
4. Demonstrate proficiency with rigid motions.
5. Determine congruence or non-congruence of different objects in a particular geometry.
6. Categorize lines in a particular geometry according to their parallelism properties.
7. Prove or disprove theorems in the various geometries.
8. Identify which familiar theorems from Euclidean geometry hold in a particular non-Euclidean setting.

Learning Activities

Nightly Homework: After most lectures, some number of exercises will be assigned for you to work on individually or with your classmates. These will be discussed at the beginning of the following class. These exercises will be fairly straightforward computations and proofs based on the material covered in the lecture and in the text. These exercises will not be graded, but successful completion of the assignments will help prepare you for examinations.

Problem Sets: Four challenging problem sets will be assigned during the course of the semester. These problems will require more thought on your part than the nightly exercises. It is not expected that every student will be able to do every problem.

Your grade will be based upon the number of problems successfully completed. The solutions must be both mathematically correct and properly written to earn credit. If you wish to show me drafts of problems before the due date, I will give you feedback on your work. No problems will be accepted after the due date.

I encourage you to consult other geometry texts in the library and to discuss the problems with your classmates. However any relevant text or conversation should be acknowledged in your write-up.

Oral Presentations: Each student will give a 10 – 15 minute presentation during the semester, explaining an assigned theorem from Euclidean geometry. You will grade one another on your presentations. Your presentation grade will be an average of the grade that I give you and the average grade given to you by classmates.

Classroom Participation: Each student is expected to participate regularly in classroom discussions.

Assessment

Examinations: There will be one mid-semester examination and one final examination. The mid-term examination is tentatively scheduled for Monday, October 13. Any change to this date will be announced at least one week before the examination. No make-up exam will be given without a doctor's excuse. The final examination will be scheduled by the registrar.

Grading: Your final grade in the course will be based upon your performance on problem sets (10% each), your oral presentation and classroom participation (10%), the mid-term examination (15%), and the final examination (35%).

Course Outline

We will begin the course with a discussion of Euclidean geometry, emphasizing the axiomatic approach of Euclid and exploring his postulates. We will then introduce non-Euclidean geometry from an axiomatic viewpoint. This discussion will encompass Chapters 1 and 10 of the text.

The next part of the course will focus on geometry from a more algebraic viewpoint. We will discuss rigid motions in Euclidean and non-Euclidean geometry, covering Chapters 2 and 3 of your text.

The third portion of the course will pursue geometry from an analytic perspective. We will develop hyperbolic geometry using a metric, covering Chapters 4 – 7 of the text.

Given time, at the end of the course, we will include topics chosen jointly by you and by me from Chapters 8, 9, 11, 12 and/or 16.