I. Basic Course Information
MAT 106: Mathematical Structures and Algorithms for Educators II is primarily a freshman level course. It is scheduled for two 80-minute meetings each week. Its prerequisite is MAT 105. It is the second course in a two-course sequence that is especially appropriate for elementary, early childhood, and special education majors.

II. Learning Goals
MAT 106, like its predecessor MAT 105, will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint. In MAT 106, students will engage in a thorough development of geometry, measurement, data analysis, statistics, and probability. Through this process, they will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. In addition, upon completion of this course, we expect students to show an improved ability to communicate mathematical ideas appropriately using the language of mathematics, to reason mathematically, to solve various types of problems using appropriate strategies, and to relate mathematics to other subjects, its applications in society, and to other mathematical topics.

These content and performance goals for MAT 106 address the goal of the Elementary and Early Childhood Department that deals with subject matter knowledge. This goal states that students should “have a deep understanding of the academic content of the subjects they teach…” MAT 105 and MAT 106 directly address this goal for mathematics. The Special Education program has similar goals dealing with subject matter knowledge.

The profound understanding of fundamental mathematics that students gain in MAT 105 and MAT 106 will prepare them for the methods course that they take in their junior year, where they learn methods and strategies for teaching elementary mathematics to various types of learners.

III. Student Assessment
Students will receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future mathematics methods courses and field placements will be used to assess the success of MAT 106 in achieving its learning goals and its contribution to the fulfillment of the Elementary and Early Childhood and Special Education program goals.

IV. Learning Activities
In the mathematics methods course that students take in the junior year, they will learn a variety of teaching strategies appropriate for teaching elementary mathematics. They will learn about state and national standards for K-8 mathematics, and how to implement these standards in the classroom.

Students must experience standards-based teaching and learning in order to understand how to implement it. Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, written homework assignments, and
group or individual projects. It is important that a variety of strategies and methods of instruction should be used to model effective teaching of mathematics.
A sample syllabus for MAT 106 follows this department syllabus. Any syllabus for MAT 106 should include the points listed below and use the suggested outline found in Part IV as a basis for decisions on the course content.

I. Basic Information
   A. Purpose statement: In 2001, the Conference Board of Mathematical Sciences, in conjunction with the American Mathematical Society and the Mathematical Association of America, the nation’s two primary national mathematical organizations, published a report entitled, *The Mathematical Education of Teachers*. Among its recommendations are:
      - Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.
      - Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.
   MAT 106 is the second half of an experience that is aimed at meeting these recommendations.
   B. Course description: This course concerns the development of geometry, measurement, data analysis, statistics, and probability. The student will be required to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality, using a variety of representations of mathematical concepts and procedures. Physical materials and models will be used to explore fundamental concepts of geometry, measurement, data analysis, statistics, and probability. This course is especially appropriate for those students preparing to be elementary, early childhood, or special education teachers.
   C. Course prerequisites: MAT 105 (Mathematical Structures and Algorithms for Educators I). This course will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint, building on knowledge that students bring with them from their K-12 education.

II. Learning Goals
   A. Content goals: Students will engage in a thorough development of geometry, measurement, data analysis, statistics, and probability. Through this process, they will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques.
   Many students will enter the course having had very little experience with probability. This is now a subject that is part of the standards at the state and national levels. Students need to have a strong understanding of this subject and its relationship to statistics before they can teach it. Many students also have misconceptions about geometry and a lack of spatial
reasoning ability. Through careful consideration of geometrical concepts and activities to develop spatial reasoning, students will feel more comfortable and knowledgeable with this subject.

B. Performance goals: By the completion of the course, the successful student will be able to demonstrate all of the following:

- Understanding of the processes and algorithms, and the purposes beneath them, found in the elementary mathematics topics mentioned above in Part A.
- An improved ability to communicate mathematical ideas appropriately using the language of mathematics.
- An improved ability to reason mathematically.
- A willingness and ability to solve various types of mathematical problems using appropriate strategies.
- Knowledge of the relationship of mathematics to other subjects, its applications in society, and relationships within mathematics itself.
- An appreciation of the history, structure, and application of mathematics.

III. Student Assessment

A. Assessment plan: Students will be assessed and receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and in-class examinations.

B. Rationale: Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Written and oral communication will directly assess some of the performance goals listed above. Group and individual projects and explorations provide students with the opportunity to explore a concept more deeply, and at the same time, assess many of the performance goals listed above. Finally, examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.

C. Methods and criteria: A syllabus should coincide with the assessment plan in Part A and clearly describe the schedule for these assessment tools, the criteria that will be used to evaluate student performance, and how grades will be calculated.

IV. Learning activities

A. Summary of learning activities: Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, written homework assignments, and group or individual projects. Outside of class, students are expected to do a
significant amount of individual or group homework to achieve the learning goals.

B. Calendar or outline: The following is a suggested guide to the organization of course topics.

- **Data Analysis and Statistics (Weeks 1-2)**
  - Representing and Interpreting Data: understanding the kinds of questions that can be addressed by data, creating data sets, displaying data.
  - Describing data: shape, spread, and center, using different forms of representation, comparing two sets of data.
  - Drawing conclusions: choosing among representations and summary statistics to communicate conclusions, understanding variability, understanding some of the difficulties that arise in sampling and inference.

- **Probability (Weeks 3-4)**
  - Making judgments under conditions of uncertainty, measuring likelihood, becoming familiar with the idea of randomness.
  - Principles of counting, empirical and theoretical probability, simulations.

- **Geometry (Weeks 5-10)**
  - Visualization skills: becoming familiar with projections, cross-sections, and decompositions of common two- and three-dimensional figures; representing three-dimensional objects in two dimensions and constructing three-dimensional objects from two-dimensional representations.
  - Basic shapes, their properties, and relationships among them: developing an understanding of lines and angles, transformations, symmetry, tessellations, congruence, and similarity.

- **Measurement (Weeks 11-14)**
  - The process of measurement: understanding the idea of a unit and the need to select a unit appropriate to the attribute being measure, understanding that measurements are approximate and that different units affect precision, being able to compare units and convert measurements from one unit to another.
  - Length, area, and volume: deriving measurement formulas for basic shapes; understanding the independence of area and perimeter, surface area and volume.
C. Rationale: By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the fundamental ideas of elementary grade mathematics and contribute to the learning goals of the course.
We will cover Chapters 7-10 in this course.

**Description of Course:**
This course concerns the development of geometry, measurement, data analysis, statistics, and probability. You will be required to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality, using a variety of representations of mathematical concepts and procedures. Physical materials and models will be used to explore fundamental concepts of geometry, measurement, data analysis, statistics, and probability. This course is especially appropriate for those students preparing to be elementary, early childhood, or special education teachers.

**Learning Goals**
Content goals: We will engage in a thorough development of geometry, measurement, data analysis, statistics, and probability. Through this process, you will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. You may enter the course having had very little experience with probability. This is now a subject that is part of the standards at the state and national levels. You need to have a strong understanding of this subject and its relationship to statistics before you can teach it. Many students also have misconceptions about geometry and a lack of spatial reasoning ability. Through careful consideration of geometrical concepts and activities to develop spatial reasoning, you will feel more comfortable and knowledgeable with this subject.

Performance goals: By the completion of the course, the successful student will be able to demonstrate all of the following:
- Understanding of the processes and algorithms, and the purposes beneath them, found in the elementary mathematics topics mentioned above.
- An improved ability to communicate mathematical ideas appropriately using the language of mathematics.
- An improved ability to reason mathematically.
- A willingness and ability to solve various types of mathematical problems using appropriate strategies.
- Knowledge of the relationship of mathematics to other subjects, its applications in society, and relationships within mathematics itself.
- An appreciation of the history, structure, and application of mathematics.

**Course Requirements:**
3 tests and/or problem sets 45%
reflections and class participation 5%
group project 5%
homework problems and other assignments 15%
final exam 30%

**Tests and/or problems sets:**
See course outline for dates. Make-ups for tests will be given only in the case of an excused absence and only if I am notified within 24 hours of the missed exam. Make-ups will generally be harder. The final exam will be given during final exam week at the scheduled time and will be comprehensive.
Homework problems:
Homework will be assigned frequently. It will not always be collected and graded. Homework may be
discussed at the beginning of class. I will tell you when a problem(s) or other assignment is to be turned in
for a grade. If you must miss class when an assignment is due, you must contact me to make other
arrangements. You are still expected to have the assignment turned in before the next class. After that, it
is considered late. Homework will only be accepted late in the case of an excused absence. For each day a
problem solution is late, it will drop a level on the rubric. Problem solutions will be graded according to
the following rubric:

**Problem solution rubric:**

Understanding the problem
- 0: No understanding of the problem or mathematical concepts
- 1: Partial understanding of problem and/or mathematical concepts
- 2: Complete understanding of problem and mathematical concepts

Methods of solution
- 0: No attempt or totally inappropriate method
- 1: Reasonable method of solution is indicated, but not well developed
- 2: Method of solution is acceptable
- 3: Method of solution is appropriate and fully developed

Getting an answer
- 0: No answer or wrong answer
- 1: Partially correct answer, computational or copying error
- 2: Correct answer

Explanation
- 0: No explanation
- 1: Unclear or insufficient explanation
- 2: Explanation lacks detail
- 3: Clear, coherent explanation

Reflections:
The reflections are meant to be a record of your thinking throughout the semester. I may give you a
question to write about before the next class. If not, you are to write about your thoughts from class; for
example, you could write about an “aha” moment you had, a concept you are still struggling with, or a
question you might have. You are not to just write a summary of the lesson! They will be collected at the
beginning of each class. The reflections will be graded using the following scale:

- 2: Complete response to any questions assigned and good record of thinking maintained for each
class.
- 1: Partial response to questions and/or no record of thinking, just summary of class.
- 0: No response and/or missing entries.

Obviously, this will be difficult to do when you miss class. Therefore, I will drop 1 or 2 of these grades at
the end of the semester. However, if you miss class, you need to find out from someone what we did
anyway, so you may still want to do a reflection for that class you missed so that I can assess your
understanding of what you missed. If you choose to do that, please turn it in to me by the next class after
the one you missed.

Learning activities:
Learning activities will consist of a combination of lectures, explorations, group work, participation in class
discussions, readings, written homework assignments and reflections, and group projects.

Group work:
A major part of this course will be spent working in small groups. We will form these groups near the
beginning of the semester with the option at some point during the semester to switch them around.
Naturally, if anyone has a major problem working in a particular group, you may tell me and we will switch
the groups. In addition to working together in class, you are encouraged to work together outside of class
on problems. However, the final expression, the answer to a question, the solution and explanation to a
problem, are matters for individual action to show that each person comprehends the matter at hand in their
own way. Work that is merely copied will not be tolerated.
## Tentative Course Outline

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
<th>Reading</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Representing and Interpreting Data</td>
<td>Section 7.1</td>
</tr>
<tr>
<td>2</td>
<td>Distributions: Center and Spreads</td>
<td>Section 7.2</td>
</tr>
<tr>
<td>3</td>
<td>Concepts Related to Chance</td>
<td>Section 7.3</td>
</tr>
<tr>
<td>4</td>
<td>Counting and Chance</td>
<td>Section 7.4</td>
</tr>
<tr>
<td>Feb. 16</td>
<td>Test 1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Basic Concepts of Geometry</td>
<td>Section 8.1</td>
</tr>
<tr>
<td>6</td>
<td>Classifying Two-Dimensional Figures</td>
<td>Section 8.2</td>
</tr>
<tr>
<td>7</td>
<td>Classifying Three-Dimensional Figures</td>
<td>Section 8.3</td>
</tr>
<tr>
<td>8</td>
<td>Congruence Transformations</td>
<td>Section 9.1</td>
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<tr>
<td>9</td>
<td>Similarity</td>
<td>Section 9.3</td>
</tr>
<tr>
<td>March 29</td>
<td>Test 2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Symmetry and Tessellations</td>
<td>Section 9.2</td>
</tr>
<tr>
<td>11</td>
<td>Systems of Measurement</td>
<td>Section 10.1</td>
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<td>12</td>
<td>Perimeter and Area</td>
<td>Sections 10.2</td>
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<tr>
<td>13</td>
<td>Surface Area and Volume</td>
<td>Section 10.3</td>
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<tr>
<td>April 22</td>
<td>Test 3 or Problem Set due</td>
<td></td>
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<tr>
<td>14</td>
<td>Review</td>
<td></td>
</tr>
<tr>
<td>Week of May 3</td>
<td>Final Exam</td>
<td></td>
</tr>
</tbody>
</table>

**SOCS**

SOCS, which stands for “Simple Online Courseware System”, is a web-based course management system. Through SOCS, you can email classmates, submit assignments through a virtual drop box, and find class resources such as this grading policy and tentative course outline, and assignments (you may need to click on the + symbol next to the heading). To get to SOCS, use any web browser and point it to the following location:

[http://socs.tcnj.edu](http://socs.tcnj.edu)

On the login page, you can find instructions for using the system under “Student Documentation”. Use your e-mail login name and password to get in. Once in SOCS, there will be a menu for this course. The departmental syllabus for this course is available on SOCS.