Program Cover Document—MAT 326: Differential Equations

I. Basic Course Information

MAT 326 is a junior-level course. It will have two 80-minute meeting periods each week and an additional one hour for recitation/laboratory work. The prerequisite is MAT 128. Co-requisite or prerequisite MAT 205 or PHY 306

The subject of Differential Equations is the study of deterministic models of nature and historically it was developed principally for study of problems of physics. Now, however, differential equations have a much broader application to chemistry, biology and the social sciences. Much modern theory of Differential Equations overlaps with statistical methods in the area of dynamical systems. A solution to a differential equation that models a physical phenomenon represents a statistical average for a statistical approach to the same physical event. Students will draw upon their knowledge of calculus and linear algebra in formulating their solutions to many types of differential equations.

II. Learning Goals

The learning goals are threefold for the student. The first goal is to teach the student mathematical techniques for solving a wide variety of differential equations. This forms a knowledge base required for successful completion of the next two learning goals.

The second goal is to have the student develop an intuitive feeling for the approach, creation and formulation of a mathematical model that addresses a problem in nature. In particular, students should emerge from this course with an understanding of the importance of mathematical applications to all of science.

The third learning goal is that of developing in the student more mathematical sophistication as well as the ability to use current technology creatively as an aid in fashioning both the mathematical model and its solution. Software packages will be used for students to better appreciate the course material.

III. Student Assessment

Students will receive regular feedback on their work through homework, quizzes, projects and examinations. Certain homework problems may be assigned via TCNJ Webwork system that provides instant feedback to students and assesses student progress and understanding of the course material. Peer reviews and student evaluations will also be used to evaluate the course.

IV. Learning Activities

Learning activities will consist of combinations of lectures, quizzes, projects and computer assignments. Individual instructors will tailor their activities according to their goals and preferences. Outside of class, students are expected to do a significant amount of individual and in some cases group work to achieve learning goals. Giving students several ways to learn and do mathematics promotes a deeper and more widely-ranging understanding of mathematics and its significance.

Departmental Course Syllabus
MAT 326: Differential Equations

I. Basic Information on Course and Instructor

A. Purpose Statement: Differential Equations provides a student with the methods of modeling nature deterministically and serves as one of the primary reasons for the study of calculus and linear algebra. Historically the calculus was invented to solve those differential equations arising from planetary studies. Students will be taught to use abstract reasoning to formulate mathematical models and then solve these equations analytically and, in many cases, with the use of technology.

B. Course Description: This course is an introduction to differential equations and in particular their applications to real world phenomena. Selected topics include: first order ordinary differential equations, linear differential equations with constant coefficients, simultaneous linear, constant coefficient differential equations, dynamical systems and Laplace transforms. Current technology such as *MATLAB*, *Mathematica* and *ODE Architect* can be used to explore the more subtle features of the solutions.

C. Course Prerequisite: MAT 128; Co-requisite: MAT 205 or PHY 306

II. Learning Goals

A. Content Goals: Students will acquire proficiency in solving a wide variety of differential equations using many different methods and approaches. Students will develop an appreciation and understanding of how mathematical models describe real-world phenomena, physical or otherwise. Optional topics, possibly covered by assignment of special projects at the discretion of the professor, include: nonlinear autonomous systems and dynamical systems. Mathematical proofs often help students appreciate the subjects at an elevated level and will be shown in class and assigned as homework throughout the course.

B. Performance Goals: Upon completing the course, students should demonstrate competence with the ideas of forming models that are deterministic (differential equations), obtaining solutions analytically and numerically, and understanding the limitations of these solutions. They should understand what it means to interpret a solution to a differential equation or system of differential equations, that is, how a solution resolves the real-world problem that the differential equation(s) modeled.

III. Student Assessment

A. Assessment Plan: Students will receive timely feedback on their homework, laboratory assignments, quizzes, student projects and examinations. Any grading policy

should clearly describe the schedule for these assessment tools with the possible exception of quizzes and how they will be used to calculate grades.

B. Rationale: Through the use of timely feedback from homework, laboratory assignments, quizzes, student projects and examinations, students will be able to see and correct their misunderstandings and thereby improve their performance. These assessment tools are similar in manner and scope to those ways in which students will need to use their knowledge in the future. Hence these tools of assessment are appropriate to assess the accomplishment of the course's learning goals.

C. Methods and Criteria: We will use the assessment of homework, quizzes, student projects and examinations to evaluate student accomplishment of the course learning goals.

IV. Learning Activities

A. Summary of Learning Activities: Learning activities will consist of a combination of lectures, discussion of homework problems, discussion and presentation of student projects and computer project results. The choices and arrangements of these activities depend upon the individual instructor.

B. Calendar and Outline: A guide to the organization of the course and a schedule of assessment tools and quizzes will be provided to the students. Homework, quizzes and examinations will be spaced at appropriate intervals throughout the semester. It is expected that each of the major topics will be given roughly equal emphasis during the course of the semester.

c. Rationale: By giving students a variety of approaches to learning and doing mathematics, these activities should promote a deeper understanding of those ways that differential equations and modeling in general help to ameliorate problems encountered in nature. This is certainly the major learning goal of this course. An even spacing of assessment tools insures that students get timely feedback on their progress.

Course Outline and Required Course Topics

The required topics for the course are:

1. **Introduction and Basic Differential Equation Terminology**

2. **First Order Differential Equations**
    a. Existence and uniqueness of solutions
    b. Direction fields
    c. Equilibria and stability analysis
    d. Separable differential equations
    e. Linear differential equations
    f. Principle of superposition
    g. Substitutions and transformations; change of variables
    h. Euler's Method and Improved Euler's Method
    i. Exact Equations*
    j. Bifurcation analysis*

3. **Mathematical Modeling and Compartmental Analysis**
    a. Mixing Problems
    b. Population Models
    c. Mass-Spring Models*

4. **Second Order Differential Equations**
    a. Linear homogeneous differential equations with constant coefficients
    b. Linear nonhomogeneous differential equations and the method of undetermined coefficients
    c. Linear nonhomogeneous differential equations and variation of parameters
    d. Linear homogeneous differential equations with variable coefficients
    e. The Reduction of order method and Cauchy-Euler equations

5. **Series Solutions of Differential Equations**
    a. Power Series Solutions of first order and second order equations near ordinary points
    b. Regular singular points and series solutions near regular singular points*

6. **Systems of Differential Equations**
    a. Linear systems – finding solutions using the elimination method*
    b. Linear systems – finding solutions using eigenvalues and eigenvectors
    c. Phase plane and classification of equilibria for systems of equations
    d. Nonhomogeneous linear systems of differential equations*
    e. Nonlinear systems of differential equations and linearization*
    f. Numerical methods for systems of differential equations*

* Indicates the optional topics to be covered if time permits.

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