

## Program Cover Sheet --- MAT 301: Number Theory

### I. Basic Course Information

MAT 301 is primarily a sophomore level course in number theory. It will be scheduled for two 80-minute lecture periods. It has MAT 200: Discrete Mathematics as a prerequisite.

### II. Learning Goals

MAT 301: Number Theory has the dual learning goals of introducing students to the content of elementary number theory while simultaneously working and improving their ability to read mathematics and write proofs. It serves as a preparatory course in the department to prepare students for the more abstract and rigorous material found in MAT 305: Abstract Algebra, one of the department's required courses for its MATA and MATT majors.

Number Theory is one of the oldest areas of mathematics and studies the basic properties of ordinary integers (0, 1, -1, 2, -2, etc.). Problems in the subject are often easy to state and understand, but yet are often surprisingly difficult to solve. The Greeks posed many number theory problems and solved some of these. Yet, some have remained unsolved until this day. Many other problems have been studied in the intervening centuries and number theory continues to be an exciting area of mathematics. One of the additional goals of MAT 301: Number Theory is to expose students to the joys and frustrations of doing mathematics by introducing them to some very interesting number theory problems.

The choice of content for MAT 301 serves two goals. First, many of the concepts used in Number Theory are also used in a more elementary manner in MAT 305: Abstract Algebra. By providing students with concrete examples, Number Theory provides a solid foundation for students to study the more abstract material in Abstract Algebra. Secondly, a majority of the students in MAT 301 are mathematics education majors. Number Theory is a required course for MATT majors as number theory is an important subject for future teachers to know. Because of the simplicity of many of its problems, number theory has long been an example of "modern mathematics" that could be used to enrich the secondary school curriculum. It is important that future teachers have a solid foundation in number theory to be able to provide such enrichment.

The second goal of Number Theory is to improve the ability of students to reason abstractly, to read mathematics, and to write proofs. MAT 200: Discrete Mathematics and MAT 205: Linear Algebra are the initial two courses students take to learn how to do proofs. Number Theory seeks to build upon the abilities they have learned in these courses, but in the context of number theory.

### III. Student Assessment

Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future courses such as Abstract Algebra will be used to assess the success of Number Theory in achieving its learning goals and its contribution to the fulfillment of the MATA, MATT, and MATC program goals. Peer reviews and student evaluations will also be used to evaluate the course.

### IV. Learning Activities

Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. These learning activities are typical of the learning activities in the MATA, MATT, MATC programs. By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of number theory and contribute to the learning goals of these programs.

## Departmental Course Syllabus --- MAT 301: Number Theory

**Introduction:** A typical syllabus for Number Theory follows this sheet. Any syllabus for Number Theory should include the points listed below (the required course requirement sections).

### I. Basic information on course and instructor

- A. Purpose statement: Number Theory introduces students to some of the classical problems in elementary number theory. While learning the subject matter of number theory, the course will also develop a student's ability to reason abstractly, to read mathematics, and to prove theorems. Through the use of challenging problems, the course should also develop a student's problem solving ability and introduce students to the joys of mathematics. The course should also provide a firm foundation in modular arithmetic for students taking Abstract Algebra.
- B. Course description: An introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves.
- C. Course prerequisites: MAT 200.

### II. Learning goals

- A. Content goals: Students will gain acquaintance with many basic topics in elementary number theory. Students will learn about primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity. Other optional topics such as sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves may be included at an instructor's discretion.
- B. Performance goals: At the completion of the course, students should demonstrate competence with number theory concepts. A successful number theory student should be able to do modular arithmetic and know how to use modular arithmetic to study Diophantine equations. They should know basic procedures to determine whether or not a Diophantine equation has solutions. They should understand the definition of a prime, their central role in arithmetic problems and be knowledgeable about primes in number systems other than the integers. They should also be proficient at finding and working with primitive roots. Students should possess improved reasoning and proof-writing ability and should exhibit a more mature ability with proofs than would normally be expected of a student at the completion of MAT 205. In particular, they should be comfortable with important techniques of proofs such as Fermat's theory of descent.

### III. Student assessment

- A. Assessment plan: Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. A syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.
- B. Rationale: Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.
- C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to use their knowledge in the future of and are an appropriate way to assess the accomplishment of course learning goals.

### IV. Learning activities

- A. Summary of learning activities: Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend

upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals.

- B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester. As a general rule, it is expected that each of the major topics of primes, unique factorization, congruences, divisibility, Diophantine equations, primitive roots, and quadratic reciprocity will be given equal emphasis during the course of the semester.
- C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper number theory understanding and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students continual regular feedback on their work.

## **MAT 301 - Number Theory**

### **Spring 2015**

#### Section 01

M/Th: 2:00pm - 3:20pm -- SCP 229

W: 4:00pm - 4:50pm -- SCP 201/SCP 224

#### Section 02

M/Th: 3:30pm - 4:50pm -- SCP 229

W: 4:00pm - 4:50pm -- SCP 201/SCP 224

**Instructor:** Steffen Marcus

**Office:** SCP 245

**Email:** marcuss@tcnj.edu

**Office Hours:**

M: 10:00am - 11:00am

W: 3:00pm - 4:00pm

#### **Course Description:**

MAT 301/Number Theory

1 course unit

*Prerequisite:* MAT 200

This course is an introduction to number theory. Topics include divisibility, primes, unique factorization, Diophantine equations, congruences, and quadratic reciprocity. Optional topics may include sums of squares, number-theoretic functions, continued fractions, prime number theory, public-key encryption, and elliptic curves. This course includes a weekly lab component where we will be writing and executing cryptographic computer algorithms using the modular arithmetic techniques studied in lecture.

Please refer to the departmental [program cover sheet and syllabus](#) for this course.

**Website:** All course information, announcements, and grades will be posted on this Canvas portal website. It also contains information that may help you succeed in this course. I'll assume you are keeping up to date with its contents.

**Course Materials and Text:** Strayer, *Elementary Number Theory*, ISBN 9781577662242

**Course Purpose and Learning Goals:** The main goal of the course is to provide an introduction to elementary and contemporary number theory. It will include many examples of intermediate and advanced techniques, methods of proofs, and rigorous analytic thinking used in upper level mathematics. In particular, the hope is that you will learn how to use your understanding of mathematics and maturity in proof writing to study a variety of interesting mathematical topics, including:

- divisibility,
- quadratic reciprocity,
- Diophantine equations,
- modular arithmetic,
- cryptography
- various other advanced topics.

MAT 301 provides a good foundation in mathematical thinking for other advanced mathematics or computer science courses.

**Course Requirements:** The course will involve a combination of lecture, group work, computer labs and in class activities. Graded components include highlighted problems, quizzes, in class projects, two in class tests, lab work, and a final exam. I will be also assigning reading from the textbook and further

recommended problems for you to think about. Mathematics is learned by consistently reinforcing the material. I will be operating under the assumption that you are reading the textbook at the pace suggested by the reading assignments, and working on the recommended problems throughout the term and not just the day before the exam.

**Evaluation:**

20% quizzes  
10% in class assignments, highlighted problems and lab work  
20% Test 1  
20% Test 2  
30% final exam

I reserve the right to change the grading scheme as I see fit. Any other grading scheme I use will only be beneficial to your grade as compared to the above standard. This course will be graded using the college's standard letter grade scale. Please refer to the college's policies regarding [final exam, evaluation, and reading days](#).

**Attendance:** I will operate under the assumption that you are attending all lectures and lab sections. I certainly think it is a really good idea to do so. My feelings will never be hurt, however, if you happen to miss a lecture. Even in the case of an emergency, you will always be responsible for the material presented in a class you may have missed. The only way to hand in homework and other written assignments is by attending class when they are due. The only way to take a quiz, test or final exam is to attend class as they are administered. Please be in touch with me immediately or as soon as possible regarding any extenuating circumstances you might have. Please also refer to the college's policies regarding [absence and attendance](#).

**Collaboration:** I never recommend doing mathematics alone. Part of the reason I love math is that it can be such a social discipline if you let it. Please study together. Work together on the recommended problems. Teach someone something you understand - it's the best way to reinforce it. That being said, when it comes time to write up any highlighted problems, *you must work alone*. The expectation is that when you are working with other people, you are talking things out and perhaps writing stuff on a blackboard or scrap piece of paper, but you are not taking personal notes. When you go back home and write your homework up in good, your work should be in your own words and not copied. You may also consult the college's policies regarding [academic integrity](#).

**Due Dates:**

Quizzes: Monday Feb 9/Feb 23/Mar 30/Apr 13; Thursday May 7  
Test 1: Monday, March 9, 2015 in class.  
Test 2: Monday, April 27, 2015 in class.  
Final Exam: TBA (arranged by university administration)

This syllabus is intended to give the student guidance in what may be covered during the term and will be followed as closely as possible. However, I reserve the right to modify, supplement and make changes as the course needs arise.

Finally, I'll refer you to the college's policies on the [americans with disabilities act](#).