Revised 7/2013

Program Cover Document --- MAT 127: Calculus A

I. Basic Course Information

MAT 127: Calculus A is a freshmen/sophomore level introductory course to single-variable calculus. It will be scheduled for three lecture periods: two periods of 80 minutes length and one one-hour meeting period. Its prerequisite is a passing grade on the departmental MAT 127 placement exam or completion of MAT 096.

II. Learning Goals

The primary emphasis of Calculus A is on learning the first-half of single-variable calculus. Its subject matter is part of the foundation that many higher-level courses in mathematics, science, and engineering are built upon.

The major topics covered are limits and derivatives, the rules of differentiation, applications of differentiation, an introduction to integrals, and applications of integrals. Upon completion of the course, we expect students to show competence with the ideas of calculus and its calculations, to understand how to apply calculus to solve real-world problems, to exhibit an improved ability to describe a real-world problem mathematically, to have an increased mathematical maturity, and to have an improved ability to read, write, and understand mathematics. These performance goals are in agreement with the department's program goals.

In Calculus A, students will gain exposure to both the theoretical and applied aspects of calculus. By working on many real-life problems, students will gain an appreciation for the practical applications of calculus. Simultaneously, their mathematical maturity will be built up through the presentation of theory and the expectation of a higher level of reasoning than has previously been demanded in their mathematics courses. They are also exposed to the concepts and techniques of problem solving through individual and group work on the exercises.

Calculus A, together with its continuation MAT 128: Calculus B, serves as a bridge course between the high-school and college mathematical curricula. College level mathematics, science, and engineering courses demand a higher level of quantitative reasoning than that demanded in the high school curriculum. As incoming students comprise the majority of enrolled students, an important goal of Calculus A is to raise the level of their mathematical reasoning skills to the collegiate level.

III. Student Assessment

Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future courses such as Calculus B will be used to assess the success of Calculus A in achieving its learning goals and its contribution to the fulfillment of the department's program goals. Peer reviews and student evaluations will also be used to evaluate the course.

IV. Learning Activities

Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals. These learning activities are typical of the learning activities in the department's programs. By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of calculus and contribute to the learning goals of these programs.

Course Syllabus Guide -- MAT 127: Calculus A

Introduction: A typical syllabus for Calculus A follows this sheet. All syllabi for Calculus A should include the points listed below and cover the items on the Calculus A Topics list with the indicated depth.

I. Basic information on course and instructor

A. Purpose statement: Calculus A, the first half of the single-variable calculus experience, is a foundational course for the mathematics, science, and engineering curricula. It introduces students both to calculus and the higher expectations of college-level mathematics courses. Calculus A should also inspire mathematical curiosity and interest in its students.

B. Course description: This course provides students with a solid grounding in single-variable calculus. The course is designed for students in the mathematical and physical sciences. Topics include functions and limits, derivatives and differentiation rules, applications of derivatives, and an introduction to integrals and their applications.

C. Course prerequisites: MAT 096 or having passed the departmental MAT 127 placement exam.

II. Learning goals

This course aims to develop student proficiency in the understanding of calculus concepts through numerical, graphical and functional analyses.

The course should be taught in a manner that develops and exhibits the following mathematical practices. Students of calculus should:

- Engage with the subject matter as they progressively grow in mathematical maturity and expertise throughout the sequence.
- Be able to address and demonstrate understanding of rigorous problems in multiple representations;
- Be able to explain and justify their solutions (including discussing them with other students),;
- Be able to apply mathematics to real-life applications;
- Be able to express their thought process and solution strategies in clear, written form.
- Be exposed to the purposeful and appropriate use of technology in the course.

A. Learning goals: MAT 127 will focus on an in depth understanding of:

a. Concept and evaluation of limits
b. Concept of continuity
c. Notions of (average and instantaneous) rate of change and computing rates of change
d. Notion of the derivative and methods of differentiation
e. Solving problems involving applications of limits and derivatives including related rates, sketching, and optimization
f. Fundamental Theorem of Calculus and Applications of Fundamental Theorem of Calculus
g. Notions of integration (Riemann sums, area under the curve)

Students will gain experience in communicating calculus topics utilizing mathematical language. The course will cover the topics listed on the attached “Calculus A Topics List” with the indicated emphases.

B. Learning Outcomes: Through solving rigorous problems and presenting written solutions clearly on formal and informal assessments, students will be able to demonstrate mastery of concepts involving:
    a. Notion of Limits and evaluation of limits (one-sided, two-sided, infinite)
    b. Continuous functions
    a. Rate of change of functions
    b. Derivatives and differentiation techniques of various functions.
    c. Applications of limits and derivatives
    d. Fundamental Theorem of Calculus and its Applications
    e. Notions of Integration including Riemann sums, and area under the curve
    f. Students should be able to demonstrate understanding of multiple (numerical, graphical and algebraic) representations of a calculus problem.
    g. Students should be able to justify their thought processes and solutions in clear, written form.
    h. Students should be able to solve a real-life problem involving calculus.

III. Student assessment

A. Assessment plan: Students will receive regular feedback on their work through the assignment of homework, quizzes, student presentations and examinations. The department has created a set of computerized homework problems (currently using the WeBWorKsystem) that all professors are required to assign to students. The amount of weight given to these homework problems is at the discretion of the instructor. The syllabus should clearly describe the schedule for these assessment tools and how they will be used to calculate grades.
B. Rationale: Students need to be able to use calculus correctly in their future courses. Through the use of regular feedback from homework, quizzes, student presentations and examinations, students will be able to see and correct their misunderstandings and improve their performance.
C. Methods and criteria: We will use the assessment of homework, quizzes, student presentations, and examinations to evaluate student accomplishment of the course learning goals. These assessment tools are similar to the manner in which students will need to display their knowledge of calculus in the future and are an appropriate way to assess the accomplishment of course learning goals.

IV. Learning activities

A. Summary of learning activities: Learning activities will consist of a combination of lectures, group work, student presentations, and computer assignments. The specific choice will depend upon the individual instructor. Outside of class, students are expected to do a significant amount of individual and group homework to achieve the learning goals.
B. Calendar or outline: A guide to the organization of the course, a schedule of assessment tools, and a plan for the coverage of topics should be provided to the students. As an approximate guide, 1-2 topics on the recommended list can be covered in a class period. Homework, quizzes, and examinations should be spaced at appropriate intervals throughout the semester.
C. Rationale By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the concepts of calculus and contribute to the learning goals of these programs. A regular spacing of assessment tools insures that students continual regular feedback on their work.

Calculus A Topics List

All listed topics are to be covered. Topics in bold should be covered in depth. The chapter numbers correspond with Stewart's *Calculus: Early Transcendentals* book.

Chapter 2: Limits and Derivatives

- 2.1 Average rate of change, Tangent and velocity problems
- 2.2 Numerical/Graphical approach to limits, **One sided limits, Infinite limits**
- 2.3 **Calculating limits using the limit laws, Algebraic manipulations to evaluate limits,** Squeeze Theorem
- 2.5 **Continuity, Intermediate Value Theorem**
- 2.6 **Limits at infinity, Horizontal asymptotes**
- 2.7 **Derivatives and rates of change, Definition of derivative at a point**
- 2.8 **Derivative as a function, Differentiability and continuity, Higher derivatives**

Chapter 3: Differentiation Rules

- 3.1 **Basic differentiation rules, Derivatives of polynomials and exponential functions**
- 3.2 **Product and quotient rules**
- 3.3 **Derivatives of trigonometric functions,** Trigonometric limits
- 3.4 **Chain rule, Derivatives of general exponential functions**
- 3.5 Implicit differentiation, Derivative of inverse functions (not done explicitly in Stewart), Derivatives of inverse trigonometric functions
- 3.6 **Derivatives of general logarithmic functions,** Logarithmic differentiation
- 3.9 Related rates
- 3.10 Linear approximation

Chapter 4: Applications of Differentiation

- 4.1 **Maximum and Miminum values including local and absolute extrema, Critical points, Optimizing on a closed interval**
- 4.2 Rolle's Theorem, Mean Value Theorem
- 4.3 How derivatives affect the shape of a graph including monotonicity of a function, **concavity and inflection points**, second derivative test for critical points
- 4.4 **Indeterminate forms and L'Hospital's Rule**
- 4.5 Summary of curve sketching, aAymptotes
- 4.7 Optimization problems
- 4.9 **Antiderivatives**

Chapter 5: Integrals

5.1 Summations, Areas and Distances
5.2 **Definite integral including approximating areas by rectangles using endpoints or midpoints**, General Riemann sums, **Definite integral as signed area, Properties of the definite integral**
5.3 **Fundamental Theorem of Calculus, part I and part II**
5.4 Indefinite integrals and the net change theorem
5.5 **Substitution rule, including change of variables for definite integrals**

Chapter 6: Applications of Integration

6.1 **Areas between curves,** including integration along the y–axis
6.2 Volume as the integral of cross-sectional area
6.4 Average value of an integrable function over closed interval.

MAT 127: Calculus A
Course Syllabus

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Office: Science Complex 207
Office Hours: M: 4-5:30, Th: 1:30-3,
and by appointment.

Books: *Calculus, Seventh Edition*, by J. Stewart.
Prerequisites: None

Course Goals: MAT 127: Calculus A is a first-year level course studying differential calculus and providing an introduction to integral calculus. The goals of the course are to provide a complete overview of the derivative and its uses, to define both the indefinite and definite integral and provide an introduction to its uses. Additional goals of the course are to familiarize you with many of the real-life applications of calculus as well as teach you some of the mathematical theory involved. Most of the course will have a computational flavor to it, but we also have proofs and more abstract material at times.

Course Content Overview: Calculus is an incredibly useful subject utilized throughout physics, chemistry, biology, and even in subjects like economics and business. Much of calculus was invented in order to solve problems arising in these discipline (for example, Newton was motivated by problems from physics such as how to model the motion of a falling apple), so we will be working with many real-world problems throughout the course. The two main subjects in our course are differential and integral calculus. These two subjects have different flavors but have many connections that one exploits in applications of calculus to the real world.

Differential calculus answers the question "How fast is thing A moving/changing?" Some things, like a car traveling on a highway at the speed limit, will travel at a constant speed like 65 mph. But when one wants to slow down, or even go faster, the car's speed is changing. "How does one calculate the car's speed when it is accelerating/decelerating?" goes to the heart of the type of questions we try to answer in differential calculus. Once one can answer it, we can plot functions, determine the flight-plan of a spacecraft, and figure out how hard and at what angle Mike Piazza has to hit a baseball for it to clear the rear fence. We can also then relate how fast a weirdly shaped bathtub (such as a bathtub for two) may fill up with water to the flow of the water into the tub. Exciting stuff, yes?

Integral calculus tries to answer the different question, "How much area is under that curve?" Until now, you have only seen how to calculate the area of simple shapes like rectangles and triangles. You may know the formula for the area of a circle, but do you understand why that formula is correct? Integral calculus will explain why $\boldsymbol{\pi r^2}$ is the correct formula for the area of a circle. The methods of integral calculus can then be used to calculate volumes of solids, lengths of curves, and surface areas of solids. All these questions are important in real life when you want to construct something and you need to know how much wood, string, paint, or cloth to buy in order to build your project. Most unexpectedly, we will be able to make use of differential calculus to help answer many of these questions.

Course and Performance Goals: The goals of the course are for you to learn calculus and appreciate its applications, to develop your ability to tackle complicated problems arising in real-life, to develop your ability to read, write, and understand proofs, and to further develop your ability to reason logically. To assist you in accomplishing these goals, we will provide you with a number of different environments in which you can do calculus. You will make mistakes. Much of the material in this course will be new to you and may take time and practice to master. The key to doing well and learning in this course is to do many exercises, to make mistakes and then to learn from them so that you do not continue making these mistakes in the future. In addition to lectures, we will have in-class individual and group work, weekly homework, out-of-class computer graded homework and quizzes to give you ample opportunities to test and develop your knowledge.

Assessment: The course grade will be an assessment of how well you understand the material of the course. Since mastery of much of the material will take time, the course grade will be heavily weighted towards the examinations at the end of the course and will be calculated using the following formula: Final Examination 40%, First In-class Exam 10%, Second In-Class Exam 20%, Homework 20%, Quizzes 10%. In addition, good/poor class participation can raise/lower your course grade by up to two-thirds of a letter grade. The professor reserves the right to change or modify this formula.

Examinations will mostly consist of questions based upon material presented in-class, and problems done either in-class, on homework, or on quizzes. As a result, it is strongly encouraged that you attend every class and that you take detailed notes. Your notes should accurately represent everything done during class. In particular, all material written on the blackboard should be faithfully and legibly recorded in your notes. One particularly effective study suggestion is to develop the habit of rewriting your notes for a class before the next class. If you should need to miss a class, please ensure that you get the notes from another student in the class. There will be no attendance policy for the lectures (there will be one for the computer labs), but in the past there has been a strong correlation between regular attendance and a good grade in this course

Homework: Weekly homework will be assigned and you will be expected to work on it in groups of two or three (turning in one copy for the whole group). The homework should represent the efforts of the group and not just one member of the group. We will form initial groups during the first week of class and change them occasionally during the course of the semester.

Computer Homework: This semester, we will be using a computerized homework system Webwork. Most weeks, we will have some homework problems that must be done online at the Webwork site. Further information will be provided in class.

Absence Policy: There will be no makeup quizzes. Makeup exams will only be given in extraordinary circumstances and only when the request has been made at least one week in advance of the exam.

Additional Help: The Office of Academic Assistance in Forcina Hall runs a mathematics help desk and has tutors in Calculus. They have both drop-in sessions and scheduled sessions.

Schedule: During the semester, we will cover Chapters 2-5 of Stewart. Should you be unsure of the material in Chapter 1, please take advantage of the Tutoring center listed above to review that material. On average we will cover one chapter every 3 weeks. The first in-class exam will be cover the material (that we cover) upto Chapter 3. and will be given around the sixth week of class. The second in-class exam will cover most of the material from Chapters 4 and the beginning part of Chapter 5 and will be given around the 10^{th} week of the course. The final examination will be cumulative.