

## **Program Cover Document --- MAT 105: Mathematical Structures and Algorithms for Educators I**

### **I. Basic Course Information**

MAT 105: Mathematical Structures and Algorithms for Educators I is primarily a freshman level course. It is scheduled for two 80-minute meetings each week. Its prerequisite is satisfaction of the basic skills requirement (see information on Records and Registration webpage at <http://www.tcnj.edu/~recreg/placementTest/skills.shtml>). It is the first course in a two-course sequence that is especially appropriate for elementary, early childhood, and special education majors.

### **II. Learning Goals**

MAT 105 and MAT 106, the second course in the sequence, will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint. In MAT 105, students will engage in a thorough development of patterns, numeration, mathematical systems, real numbers, and number theory. Through this process, they will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. In addition, upon completion of this course, we expect students to show an improved ability to communicate mathematical ideas appropriately using the language of mathematics, to reason mathematically, to solve various types of problems using appropriate strategies, and to relate mathematics to other subjects, its applications in society, and to other mathematical topics.

These content and performance goals for MAT 105 address the goals of the Elementary and Early Childhood and Special Education programs that deal with subject matter knowledge. For example, Goal 1.0 in the Report of the Department of Elementary and Early Childhood Education on the Transformative Change states that, "The teacher candidate will demonstrate proficiency in theoretical and practical aspects of all subject areas by identifying and describing the structure, major concepts, knowledge, and skills in the academic disciplines." MAT 105 and MAT 106 directly address this goal for mathematics. Learning Objectives listed for Goal 1.0 include demonstrating "the ability to use the terminology and language of a discipline to communicate the structure of that discipline." The focus on communication in MAT 105 will help students to meet this objective. Goal 2.0 deals with the ability of the teacher candidate to "demonstrate proficiency in selecting and organizing the key concepts, knowledge, and skills in the subject matter area." In order to be able to satisfy this goal, students need a strong understanding of the subject matter, including the relationships between concepts, another focus of MAT 105. The Special Education program has similar goals dealing with subject matter knowledge.

The profound understanding of fundamental mathematics that students gain in MAT 105 and MAT 106 will prepare them for the methods course that they take in their junior year, where they learn methods and strategies for teaching elementary mathematics to various types of learners.

### **III. Student Assessment**

Students will receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and examinations. Through this feedback, students will be able to see and correct their misunderstandings and improve their performance. Student performance on these assessment instruments and the performance of students in their future mathematics methods courses and

field placements will be used to assess the success of MAT 105 in achieving its learning goals and its contribution to the fulfillment of the Elementary and Early Childhood and Special Education program goals.

#### **IV. Learning Activities**

In the mathematics methods course that students take in the junior year, they will learn a variety of teaching strategies appropriate for teaching elementary mathematics. They will learn about state and national standards for K-8 mathematics, and how to implement these standards in the classroom.

Students must experience standards-based teaching and learning in order to understand how to implement it. Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, written homework assignments, and group or individual projects. It is important that a variety of strategies and methods of instruction should be used to model effective teaching of mathematics.

## Departmental Course Syllabus – MAT 105: Mathematical Structures and Algorithms for Educators I

A typical syllabus for MAT 105 follows this. Any syllabus for MAT 105 should include the points listed below and use the suggested outline found in Part IV as a basis for decisions on the course content.

### I. Basic Information

A. Purpose statement: In 2001, the Conference Board of Mathematical Sciences, in conjunction with the American Mathematical Society and the Mathematical Association of America, the nation's two primary national mathematical organizations, published a report entitled, *The Mathematical Education of Teachers*. Among its recommendations are:

- Prospective teachers need mathematics courses that develop a deep understanding of the mathematics they will teach.
- Prospective elementary grade teachers should be required to take at least 9 semester-hours on fundamental ideas of elementary school mathematics.

MAT 105 is the first half of an experience that is aimed at meeting these recommendations.

- B. Course description: This course concerns the development of number systems, algebraic structures, and algorithms. The student will be required to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality, using a variety of representations of mathematical concepts and procedures. Physical materials and models will be used to explore fundamental properties of number systems, to model algorithms, and to explore selected algebraic structures. This course is especially appropriate for those students preparing to be elementary, early childhood, or special education teachers.
- C. Course prerequisites: Satisfaction of basic skills requirement (see information on Records and Registration webpage at <http://www.tcnj.edu/~recreg/placementTest/skills.shtml>). This course will study the fundamental principles that underlie elementary school mathematics from an advanced viewpoint, building on knowledge that students bring with them from their K-12 education.

### II. Learning Goals

A. Content goals: Students will engage in a thorough development of patterns, numeration, mathematical systems, real numbers, and number theory. Through this process, they will develop understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques.

Many students will enter the course with the misconception that mathematics is all about computation and following procedures. Through careful consideration of many types of problems and alternative algorithms, students will come to understand that mathematics is much more than computation, and that there is often more than one approach to solving any problem.

B. Performance goals: By the completion of the course, the successful student will be able to demonstrate all of the following:

- Understanding of the processes and algorithms, and the purposes beneath them, found in the elementary mathematics topics mentioned above in Part A.
- An improved ability to communicate mathematical ideas appropriately using the language of mathematics.
- An improved ability to reason mathematically.
- A willingness and ability to solve various types of mathematical problems using appropriate strategies.
- Knowledge of the relationship of mathematics to other subjects, its applications in society, and relationships within mathematics itself.
- An appreciation of the history, structure, and application of mathematics.

### III. Student Assessment

- A. Assessment plan: Students will be assessed and receive regular feedback on their work through the assignment of homework, written and oral communication, group and/or individual projects and/or explorations, and in-class examinations.
- B. Rationale: Homework assignments will provide students with opportunities to attempt lengthier, more challenging problems than is possible on an examination as well as offering students practice at exam-style problems. Written and oral communication will directly assess some of the performance goals listed above. Group and individual projects and explorations provide students with the opportunity to explore a concept more deeply, and at the same time, assess many of the performance goals listed above. Finally, examinations, which normally preclude both the use of books and the practice of group discussion, enable the professor to assess the knowledge an individual student has readily available.
- C. Methods and criteria: A syllabus should coincide with the assessment plan in Part A and clearly describe the schedule for these assessment tools, the criteria that will be used to evaluate student performance, and how grades will be calculated.

### IV. Learning activities

- A. Summary of learning activities: Learning activities will consist of a combination of lectures, explorations, group work, participation in class discussions, readings, written homework assignments, and group or individual projects. Outside of class, students are expected to do a significant amount of individual or group homework to achieve the learning goals.
- B. Calendar or outline: The following is a suggested guide to the organization of course topics.
  - Problem-solving and reasoning (Week 1)
    - Introduction to problem-solving strategies and processes, including patterns as a fundamental theme in mathematics.
    - Representing and justifying general arithmetic claims, using a variety of representations; understanding different forms of argument and learning to devise deductive arguments.
    - The power of algebraic notation: developing skill in using algebraic notation to represent calculation, express identities, and solve problems.

These topics will be integrated throughout the course.
  - Structures and Concepts of Arithmetic

- Place value: how place value permits efficient representation of numbers. (Week 2)
- Operations on whole numbers, including having a large repertoire of interpretations of addition, subtraction, multiplication, and division and of ways they can be applied. Field axioms: recognizing commutativity, associativity, distributivity, identities, and inverses as properties of operations. (Weeks 3-5)
- Standard algorithms and non-standard methods for multidigit calculations: the reasoning behind the procedures and how the base-10 structure is implicated. (Weeks 3-5)
- Concepts of integers, rationals, and real numbers: what integers, fractions, and decimals are; how operations on whole numbers extend to integers and rational numbers and the mathematics that underlies the procedures; and the behavior of units under the operations. (Weeks 9-11)
- Proportional Reasoning (Weeks 12 & 13)
- Sets and Number Theory (Weeks 6-8)
  - Set theoretical concepts needed to understand number systems.
  - Fundamental properties of number systems and elementary number theory, including divisibility and the Fundamental Theorem of Arithmetic.
- Patterns and Functions (time permitting Week 14)
  - Recognizing and describing mathematical relationships; number sequences and functional relationships.

C. Rationale: By giving students a multitude of ways to learn and do mathematics, the learning activities promote a deeper understanding of the fundamental ideas of elementary grade mathematics and contribute to the learning goals of the course.

**THE COLLEGE OF NEW JERSEY**  
**Department of Mathematics and Statistics**

**Course Syllabus:** MAT105 Math. Structures and Algorithms for Educators I  
**Textbook:** *Mathematics for Elementary School Teachers* by Bassarear, 2nd edition,  
published  
by Houghton Mifflin in 2001.

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**Course Goals:** MAT105 Math. Structures and Algorithms for Educators I is the first of a two course sequence that explores elementary school mathematics from an advanced viewpoint. Its primary goal is to develop students' relational understanding of mathematical structures and the connectedness of ideas within and between concepts rather than just an instrumental understanding. This understanding comes out of student exploration and investigation and out of attempts to connect new knowledge with previously acquired knowledge. This course is especially appropriate for those students preparing to be elementary, early childhood, and special education teachers.

**Course Content and Overview:** Students will engage in a thorough development of patterns, numeration, mathematical systems, real numbers, and number theory. They will develop a deep understanding of the processes and algorithms found in elementary mathematics and discover purposes beneath the symbols and techniques. Physical materials and models will be used to explore fundamental concepts in each area.

**Performance Goals:** The student will be required to reason mathematically, solve problems, and communicate mathematics effectively at different levels of formality, using a variety of representations of mathematical concepts and procedures. These activities are consistent with the five School of Education conceptual themes, the readings, assignments, activities and assessments in this course are designed to provide the students with the knowledge, skills and understanding of: 1.) Knowledge and Inquiry 2.) Multiculturalism, Diversity, Inclusion 3.) Multiple Contexts and Communities 4.) Leadership and Advocacy 5.) Excellence in Practice

**Student Assessment:** The final grades will be based on class participation and preparation, three tests, individual and group exploratory assignments, and a final examination. You are invited to participate in class by taking notes, assisting your peers and myself during class, explaining your reasoning and problem solving methods, making brief presentations based on your explorations. You are invited to ask questions, and to challenge and discuss the ideas presented in class.

Class participation & preparation:	10%
Assignments:	25%

Tests:	45%
Final Examination:	20%

**Learning Activities:** These activities will include a combination of lectures, explorations, group work, participation in class discussions and demonstrations, readings and homework assignments.

## COURSE OUTLINE

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Week 1	Introduction, assignments, the NCTM's Five Process Standards & Problem Solving and patterns as fundamental themes in mathematics.
Week 2	The value and necessity of sets and functions in mathematical thinking, numeration, and the Alphabitia activity
Week 3	Fundamental properties of number systems
Week 4	<b>TEST I</b> , Understanding the four operations coupled with base-10 concepts
Week 5	Exploration of standard and non-standard algorithms
Week 6	Set theoretical concepts needed to understand number systems
Week 7	<b>TEST II</b> , Mental arithmetic and estimation
Week 8	Fundamental properties of number systems and elementary number theory
Week 9	Primes and composite numbers, GCF and LCM
Week 10	<b>TEST III</b> , Number concept including Divisibility and the Fundamental Theorem of Arithmetic
Week 11	Concepts related to integers, rational, and real numbers
Week 12	Explorations of decimals, fractions and percent as extensions to the set of whole numbers
Week 13	Field axioms
Week 14	Proportional Reasoning, ratio and proportion
Week 15	<b>FINAL EXAMINATION</b>